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EXISTENCE RESULTS FOR A GENERALIZED NONLOCAL ABSTRACT CAUCHY PROBLEM VIA THE ψ- HILFER DERIVATIVE

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Abstract. In this paper, we study, in an infinite dimensional Banach space $\mathbb{X}$, the existence and uniqueness of mild solutions to the Cauchy problem for the semilinear differential evolution equations with nonlocal conditions

$$^{\mathbb{H}}D^{\alpha,\beta;\psi}u(t) = A(t)u(t) + f(t,u(t)), \qquad 0 \leqslant t \leqslant T$$
$$I_{0+}^{1-\gamma;\psi}u(0) + h(u) = u_0,$$

where $^{\mathbb{H}}D^{\alpha,\beta;\psi}$ is the ψ-Hilfer operator, for $0 < \alpha < 1$, $0 < \beta < 1$, and $A(t) : \mathbb{X} \to \mathbb{X}$ is a continuous operator for each $t \in [0,T]$. The function $f : [0,T] \times \mathbb{X} \to \mathbb{X}$ is not necessarily lipschitzian. We first derive a variation of constants formula. Then we use the Wessinger and the Krasnoselskii's fixed point theorems to achieve our results. These results generalize a recent work by K. Balachandran *et al.* We provide an example to illustrate our abstract results.

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